Chapter 1

Trigonometric Levelling

1.1 INTRODUCTION

Geodetic surveying consists of precisely measuring positions on the earth's surface, of a system of widely separated points. The positions are determined both relatively, in terms of the length and azimuths of the lines joining them, and absolutely, in terms of the coordinates—latitude, longitude, and elevation above mean sea level. These points (of observation) form the control points (stations) to which topographic, hydrographic, engineering and other surveys may be referred so as to localize the error between the geodetic stations.

In geodetic surveys the horizontal control is established by precise traversing or triangulation and will be discussed in Chapter 2. To establish vertical control there are two methods: precise levelling and trigonometric levelling. Precise levelling has already been discussed in Vol. I. It is similar to spirit levelling but is carried out with greater refinement of instruments and techniques.

Trigonometrical levelling may be defined as the process of determining the relative elevations of different stations from observed vertical angles and known distances—either horizontal or geodetic lengths at mean sea level. In this method of levelling, it is desirable to locate the instrument station at the centre of the area and to observe the vertical angles outwards from this point.

The measurement of vertical angles between triangulation stations is usually undertaken at the same time as that of horizontal angles. The concurrent observation of the vertical angles alongwith the horizontal angles may prove to be economical, and for that reason it may be convenient to deal with the determination of height by the observation of vertical angles, rather than by precise levelling. However, the method of trigonometrical levelling is less accurate than precise (spirit) levelling in flat terrains. In mountainous regions and rugged terrains, the method is of great value, and the results are comparatively more accurate.

Depending upon the number of stations that can be occupied by the theodolite, there are two methods of determining the difference in elevations of two stations. When the observation is made from one of the stations, it is known as the *single* *observation method or direct method*, whereas in the other case when observations are made from both the stations, it is known as the *reciprocal observation method*. The latter one is preferred since the errors due to curvature and uncertain refraction are eliminated. Since the observations in the direct method are influenced by irregularities in the coefficient of refraction, it is desirable to make the measurement during the time of minimum refraction effect, i.e. early morning or otherwise, wherever practicable, the reciprocal method of observation should be preferred.

1.2 CORRECTION FOR CURVATURE AND REFRACTION

Let A and B be two stations and it is required to determine the true difference in their elevations (Fig. 1.1). The level lines through A and B are AA′ and BB′, respectively. AA_1 and BB_1 are the horizontal lines through A and B, respectively. H is the true difference of elevation between A and B. D is the spheroidal distance AB and d is the corresponding horizontal distance AB. θ is the angle subtended by the distance AB at the centre of the earth, γ the angle of refraction, and m the coefficient of refraction. The angle of refraction at the two stations are assumed to be equal. C is the angle of curvature and R the radius of the earth. An observer at A sights along AB_2 instead of AB. α is the angle of elevation at A and β is the angle of depression at B. Therefore, the actual observed vertical angle at A is B_2AA' while the true angle is BAA_1. Similarly at B the actual observed vertical angle is A_2BB_1 while the true angle is ABB_1. The corrected observed angles for refraction are therefore $(\alpha - \gamma)$ and $(\beta + \gamma)$. It is a usual practice to express the angle γ in terms of a fraction of angle θ, as $\gamma = m\theta$

or $$\gamma = \frac{mD}{R \sin 1''} \tag{1}$$

Now, $$\angle B_1BA = \angle BAA'' + \angle BA''A$$

or $$\beta + \gamma = (\alpha - \gamma) + \theta$$

or $$\gamma = \frac{\alpha - \beta + \theta}{2} \tag{2}$$

When both the angles are angles of depression, then

$$\gamma = \frac{\theta - \alpha - \beta}{2} \tag{3}$$

Angular correction for curvature, $C = \angle A_1AA'$. Hence,

$$C = \frac{\theta}{2} \tag{4}$$

or $$C = \frac{D}{2R \sin 1''} \tag{5}$$

$$\text{Total correction} = -\frac{mD}{R \sin 1''} + \frac{D}{2R \sin 1''}$$

$$= \frac{D}{2R \sin 1''} \times (1 - 2\text{ m}) \tag{6}$$

The combined correction is additive to an angle of elevation and subtractive to an angle of depression.

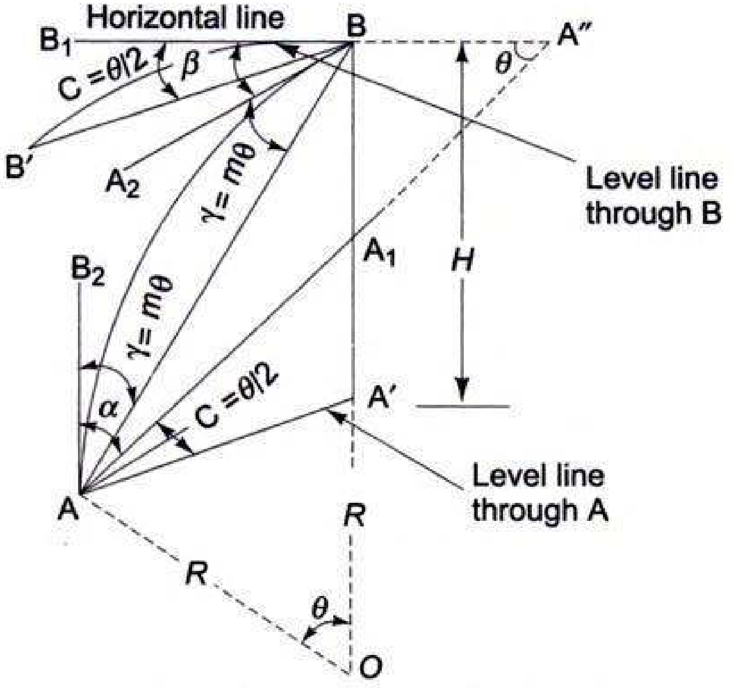

Fig. 1.1 Curvature and refraction

1.3 AXIS SIGNAL CORRECTION

Signals are used at stations to observe vertical angles. The height of signals used is usually different at various stations, which necessitates the observed vertical angles to be corrected. This is known as *axis signal correction* or *eye and object correction* or *instrument and signal correction.*

Let A and B be two stations. The vertical angle is to be observed from A to B. If the height of instrument at A is equal to the height of signal at B, the observed angle will be the true vertical angle. But, since the two heights h_1 and S_2 (Fig. 1.2) will not be same, a correction has to be applied.

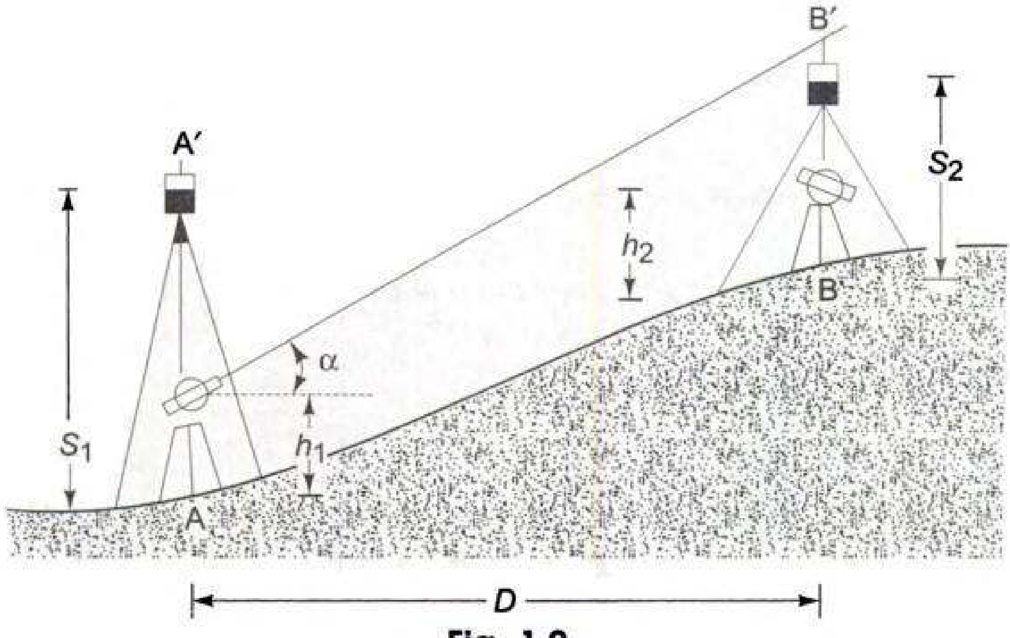

Fig. 1.2

Let h_1 be the height of instrument at A and S_2 be the height of signal at B. α is the observed vertical angle and α_1 is the corrected vertical angle for the axis signal. Let D by the horizontal distance AB (Fig. 1.2) equal to the spheroidal distance AA′ (Fig. 1.3). Axis signal correction is δ_1 ($\angle$BAE in Fig. 1.3). In triangle ABO,

$$\angle BAO = \angle A_1AO + \angle BAA_1 = 90° + \alpha$$
$$\angle AOB = \theta$$
$$\angle ABO = 180° - [(90° + \alpha) + \theta] = 90° - (\alpha + \theta)$$
$$\angle EBF = 90° - [90° - (\alpha + \theta)] = \alpha + \theta$$

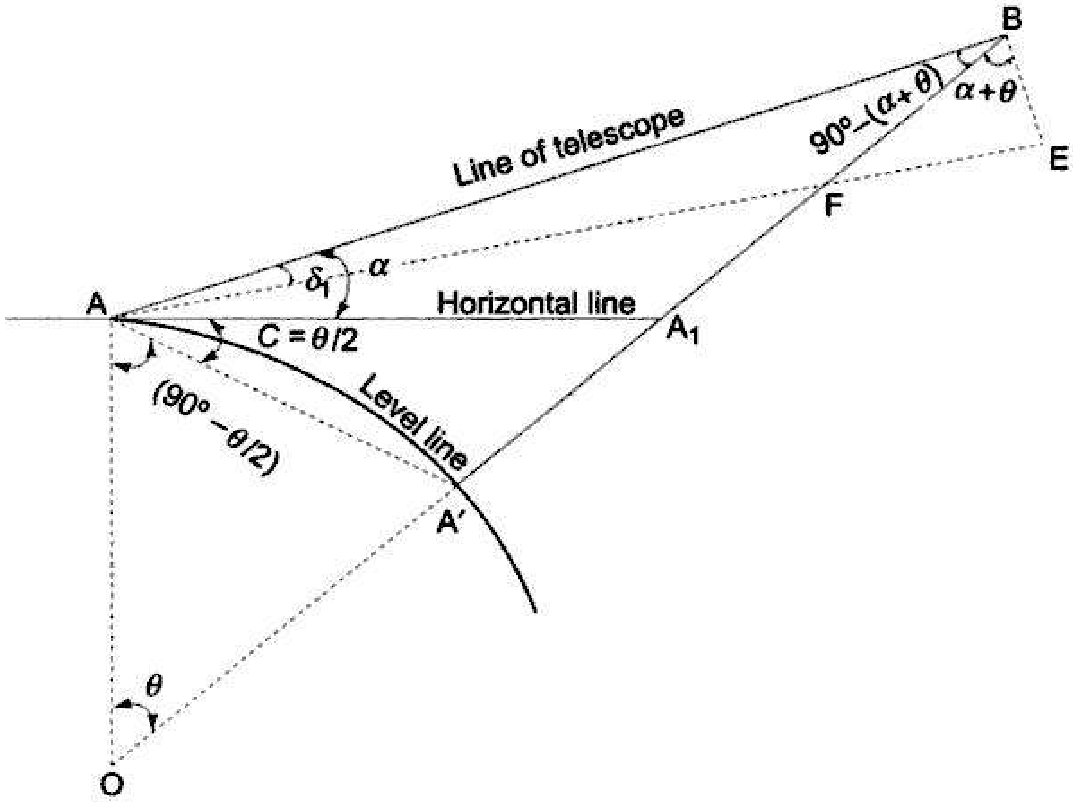

Fig. 1.3 Axis signal correction

The angle δ_1 is usually quite small and therefore angle BEF = 90°

$$BE = BF\cos(\alpha + \theta) = (S_2 - h_1)\cos(\alpha + \theta)$$

In triangle AA′B

$$\angle BAA' = \alpha + \frac{\theta}{2};\ \angle ABA' = 90° - (\alpha + \theta)$$

$$\angle AA'B = 180° - \left[(90° - (\alpha + \theta)) + \left(\alpha + \frac{\theta}{2}\right)\right] = 90° + \frac{\theta}{2}$$

By sine rule.

$$\frac{AB}{\sin \angle AA'B} = \frac{AA'}{\sin \angle ABA'}$$

or

$$\frac{AB}{\sin\left(90° + \frac{\theta}{2}\right)} = \frac{D}{\sin[90° - (\alpha + \theta)]}$$

or
$$AB = D\frac{\sin\left(90^\circ + \frac{\theta}{2}\right)}{\sin(90^\circ - (\alpha + \theta))} = D\frac{\cos\frac{\theta}{2}}{\cos(\alpha + \theta)} \quad (7)$$

From triangle ABE
$$\tan\delta_1 = \frac{BE}{AB} = \frac{(S_2 - h_1)\cos(\alpha + \theta)}{D\left[\frac{\cos\frac{\theta}{2}}{\cos(\alpha + \theta)}\right]}$$

or
$$\tan\delta_1 = \frac{(S_2 - h_1)\cos^2(\alpha + \theta)}{D\cos\frac{\theta}{2}} \quad (8)$$

Usually θ is very small as compared to α, hence the equation can be reduced to
$$\tan\delta_1 = \frac{(S_2 - h_1)\cos^2\alpha}{D} \quad (9)$$

If the vertical angle is small, the equation reduces to
$$\delta_1 = \frac{S_2 - h_1}{D\sin 1''} \quad (10)$$

Similarly, if observations are made from B towards A with β as the observed vertical angle and β_1 the corrected vertical angle for axis signal correction, it can be shown that
$$\tan\delta_2 = \frac{(S_1 - h_2)\cos^2\beta}{D}$$

☑ **Note** The correction is negative for an angle of elevation and positive for an angle of depression.

Example 1.1 A vertical angle of elevation was observed from a station P as 2°32′25″. Determine its true value if the height of instrument at P is 1.2 m and height of signal at the other station Q is 5.2 m. The two stations P and Q are 5200 m apart. Take the value of R sin 1″ as 30.88 m. The coefficient of refraction may be assumed to be 0.07. Find also the true value of the angle observed if it was an angle of depression.

Solution

Case I:

$$\alpha = 2^\circ 32' 25''$$
$$h_1 = 1.2 \text{ m}$$
$$S_2 = 5.2 \text{ m}$$
$$D = 5200 \text{ m}$$
$$m = 0.07$$

Axis signal correction,
$$\delta = \frac{S_2 - h_1}{D\sin 1''}$$
$$= \frac{5.2 - 1.2}{5200} \times 206265$$
$$= 158.66'' = 2'38.66''(-)$$

Central angle, $\theta = \frac{D}{R \sin 1''}$

$= \frac{5200}{30.88}$

$= 168.39'' = 2'48.39''$

Curvature correction, $C = \frac{\theta}{2} = \frac{168.39}{2}$

$= 84.19'' = 1'24.19''(+)$

Refraction correction, $\gamma = m\theta = 0.07 \times 168.39 = 11.78''(-)$

$\therefore$ Corrected vertical angle, $\alpha_1 = 2°32'25'' - 2'38.66'' + 1'24.19'' - 11.78''$

$= 2°30'58.75''$.

Case II:

Observed vertical angle = $-\,2°32'25''$

Axis signal correction = $2'38.66''$ (+)

Curvature correction = $1'24.19''(-)$

Refraction correction = $11.78''(+)$

The correct vertical angle = $-\,2°32'25'' + 2'38.66'' - 1'24.19'' + 11.78''$

$= -\,2°30'58.7''$

1.4 DIFFERENCE OF ELEVATION OF TWO STATIONS BY SINGLE OBSERVATION

When it is not possible to occupy both the stations, one of them being inaccessible, this method of determining the difference in elevation between the two stations is used. The vertical angle observed is corrected for curvature and refraction. Since the coefficient of refraction varies with temperature, this method does not yield accurate results. The effect of refraction is opposite to that of curvature tending to increase or decrease the vertical angle according to whether the angle is of elevation or depression.

1.4.1 Approximate Expressions

In Fig. 1.4, let A and B be the two stations and it is required to determine the difference of elevation between them. It may be computed by observing the vertical angle from any one of these stations, say A, and if the spheroidal distance D between these stations is known. The observed vertical angle is α (angle of the elevation), the angle of refraction is γ and the angle of curvature is C. Assuming AA′B as a plane right angled triangle, an approximate expression for height difference can be developed. Difference in height of A and B = A′B = $H = D \tan \phi$

where $\phi = \alpha + (C - \gamma)$

$$\therefore \quad H = D \tan [\alpha + (C - \gamma)] \tag{11}$$

or $H = D \tan [\alpha + (\theta/2 - m\theta)]$

$$\text{or} \quad H = D \tan \left[\alpha + (1 - 2m)\frac{D}{2R \sin 1''}\right] \tag{12}$$

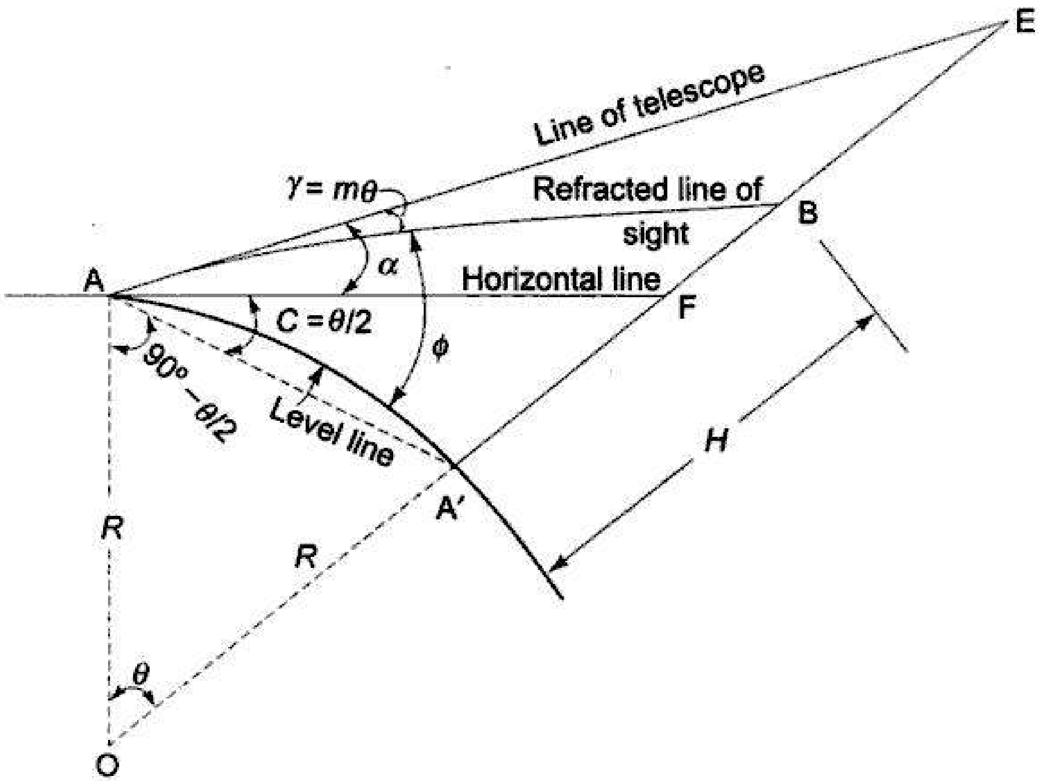

Fig. 1.4 Single observation method (angle of elevation)

In case if the vertical angle (β) observed is the angle of depression, as at station B (Fig. 1.5) the approximate expression for difference in elevation, will be the difference in heights of B and A

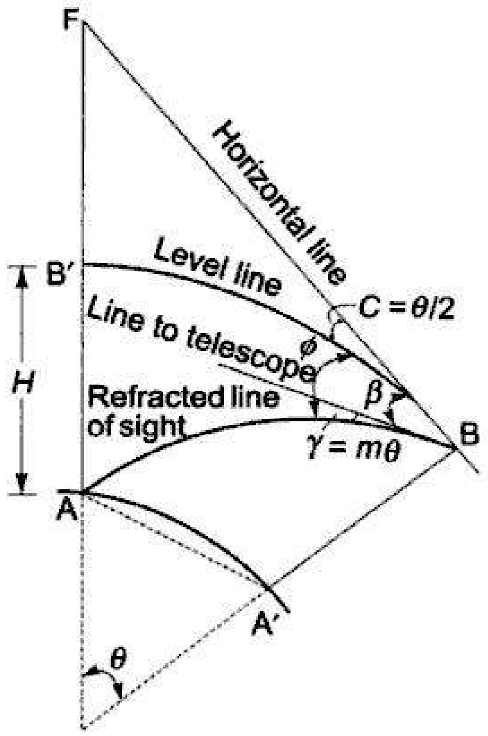

Fig. 1.5 Single observation method (angle of depression)

$$B'A = H = D \tan \phi$$

where $\phi = \beta - C + \gamma = \beta - (C - \gamma)$

or $$H = D \tan [\beta - (C - \gamma)]$$

or $$H = D \tan \left[\beta - \left(\frac{\theta}{2} - m\theta\right)\right]$$

or $$H = D \tan \left[\beta - (1 - 2m) \frac{D}{2R \sin 1''}\right] \tag{13}$$

1.4.2 Exact Expressions

In Fig. 1.4, α is the observed vertical angle of elevation, D is the horizontal distance AA′ and let it be equal to the spheroidal distance AA′. Let α_1 be the angle corrected for axis signal

$$\angle BAA' = \alpha + (C - \gamma) = \alpha + \frac{\theta}{2} - m\theta$$

$$\angle AA'B = \left(90° - \frac{\theta}{2}\right) + \theta = 90° + \frac{\theta}{2}$$

$$\angle ABA' = 180° - \left(\alpha + \frac{\theta}{2} - m\theta\right) - \left(90° + \frac{\theta}{2}\right) = 90° - (\alpha + \theta - m\theta)$$

Applying sine rule

$$\frac{BA'}{\sin BAA'} = \frac{AA'}{\sin ABA'}$$

or $$\frac{H}{\sin\left(\alpha + \frac{\theta}{2} - m\theta\right)} = \frac{D}{\sin [90° - (\alpha + \theta - m\theta)]}$$

or $$H = D \frac{\sin(\alpha + \theta/2 - m\theta)}{\cos(\alpha + \theta - m\theta)}$$

We know that, $D = R\theta$

or $$\theta = \frac{D}{R} = \frac{D}{R \sin 1''}$$

$\therefore$ $$H = D \frac{\sin\left(\alpha + \frac{D}{2R \sin 1''} - \frac{mD}{R \sin 1''}\right)}{\cos\left(\alpha + \frac{D}{R \sin 1''} - \frac{mD}{R \sin 1''}\right)}$$

or $$H = D \frac{\sin\left(\alpha + (1 - 2m) \frac{D}{2R \sin 1''}\right)}{\cos\left(\alpha + (1 - m) \frac{D}{R \sin 1''}\right)} \tag{14}$$

Equation (14) can be modified by replacing α by α_1.

In Fig. 1.5, β is the observed vertical angle of depression, β_1 is the angle corrected for axis signal

$$\angle ABB' = \beta - C + \gamma$$

$$= \beta - \frac{\theta}{2} + m\theta$$

$$\angle BB'A = (90° - \theta) + \frac{\theta}{2}$$

$$= 90° - \frac{\theta}{2}$$

$$\angle B'AB = 180° - (90° - \theta/2) - (\beta - \theta/2 + m\theta)$$

In triangle BAB′

$$\frac{AB'}{\sin ABB'} = \frac{BB'}{\sin B'AB}$$

or $$\frac{H}{\sin\left(\beta - \frac{\theta}{2} + m\theta\right)} = \frac{D}{\sin[90° - (\beta - \theta + m\theta)]}$$

or $$H = D\,\frac{\sin(\beta - \theta/2 + m\theta)}{\cos(\beta - \theta + m\theta)}$$

or $$H = D\,\frac{\sin\left[\beta - (1 - 2m)\dfrac{D}{2R\sin 1''}\right]}{\cos\left[\beta - (1 - m)\dfrac{D}{R\sin 1''}\right]} \qquad (15)$$

Equation (15) can be modified by replacing β by β_1.

Example 1.2 Two triangulation stations A and B are 3200.65 m apart. Find the difference of elevation of the two stations for the following data:

Angle of depression at B to A	=	2°18′16″
Height of signal at A	=	4.23 m
Height of instrument at B	=	1.24 m
Coefficient of refraction at B	=	0.07
R sin 1″	=	30.88 m
R.L. of B	=	242.6 m

Solution

Axis signal correction $\delta = \dfrac{S_2 - h_1}{D\sin 1''}$

$$= \frac{4.23 - 1.24}{3200.65} \times 206265$$

$$= 192.69'' = 3'12.69''\ (+)$$

Vertical angle corrected for axis signal, $\beta_1 = 2°18'16'' + 3'12.69''$

$$= 2°21'28.69''$$

$$\text{Central angle, } \theta = \frac{D}{R \sin 1''}$$

$$= \frac{3200.65}{30.88} = 103.64'' = 1'43.64''$$

$$\text{Curvature correction, C} = \frac{\theta}{2} = \frac{103.64''}{2} = 51.82''$$

$$\text{Refraction correction, } \gamma = m\theta = 0.07 \times 103.64 = 7.25''$$

$$H = D\,\frac{\sin(\beta_1 + m\theta - \theta/2)}{\cos(\beta_1 + m\theta - \theta)}$$

$$= \frac{3200.65 \sin(2°21'28.69'' + 7.25'' - 51.82'')}{\cos(2°21'28.69'' + 7.25'' - 1'43.64'')}$$

$$= 131.12 \text{ m}$$

$\therefore$ R.L. of A $= 242.6 - 131.12$

$= 111.48$ m

Example 1.3 Two triangulation stations A and B are 2800 m apart. Observations were made for vertical angle of elevation from A to B and the mean angle observed was 1°28′32″. The height of the instrument was 1.38 m and the signal was 2.46 m high. If the reduced level of station A was 125 m and the coefficient of refraction was 0.07, calculate the reduced level of B. The radius of the earth is 6372 km.

Solution

$$\text{Axis signal correction} = \frac{2.46 - 1.38}{2800} \times 206\,265$$

$$= 1'19.56''$$

$$\text{Vertical angle corrected for axis signal, } \alpha_1 = 1°28'32'' - 1'19.56''$$

$$= 1°27'12.44''$$

$$\text{Central angle, } \theta = \frac{D}{R \sin 1''}$$

$$= \frac{2800}{6372 \times 10^3} \times 206\,265$$

$$= 90.637''$$

$$\text{Correction for curvature, C} = \frac{\theta}{2} = \frac{90.637''}{2} = 45.318''$$

$$\text{Correction for refraction, } \gamma = m\theta = 0.07 \times 90.637'' = 6.34''$$

$$H = D\,\frac{\sin(\alpha_1 - m\theta + \theta/2)}{\cos(\alpha_1 - m\theta + \theta)}$$

$$v = \frac{2800 \sin(1°27'12.44'' - 6.34'' + 45.318'')}{\cos(1°27'12.44'' - 6.34'' + 90.637'')}$$

$= 71.578$ m

∴ The reduced level of B = 125 + 71.578

= 196.578 m

1.5 DIFFERENCE OF ELEVATION OF TWO STATIONS BY RECIPROCAL OBSERVATIONS

In this method of finding out the difference of elevation of two stations, the observations are made simultaneously from both the stations to eliminate the effect of refraction completely. As far as possible, the observations are made at the time of minimum refraction, i.e. between 10 a.m. and 3 p.m. The mean of the vertical angles is obtained and used for calculating the difference in elevation.

In Fig. 1.6, A and B are the two stations and the difference of elevation of these is required. α and β are the vertical angles observed simultaneously at the two stations. The angle of refraction is γ and the angle of curvature is C at the two stations. Let the corrected vertical angles for signal be α_1 and β_1.

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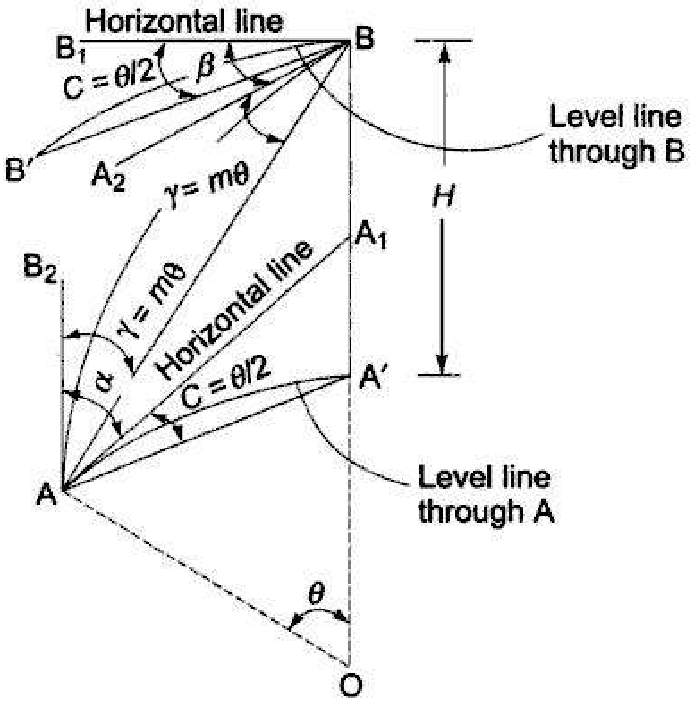

Fig. 1.6 Difference in elevation by reciprocal observation

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The mean vertical angle $= \dfrac{\alpha_1 + \beta_1}{2}$ (16)

$$\angle BAA' = \left[\alpha + \left(\frac{\theta}{2} - m\theta\right)\right]$$

The sign of correction being positive for angle of elevation.

$$\angle ABB' = \left[\beta - \left(\frac{\theta}{2} - m\theta\right)\right]$$

The sign of correction being negative for angle of depression. Since the chords AA′ and BB′ are parallel, ∠BAA′ = ∠ABB′

$$\therefore \quad \alpha + \left(\frac{\theta}{2} - m\theta\right) = \beta - \left(\frac{\theta}{2} - m\theta\right) = \frac{\alpha + \beta}{2} \tag{17}$$

In triangle AA′B

$$AA' = D \text{ (the spheroidal distance)}$$

$$\angle BAA' = \alpha + \frac{\theta}{2} - m\theta$$

$$\angle ABA' = 90° - (\beta + m\theta)$$

By sine rule

$$\frac{BA'}{\sin BAA'} = \frac{AA'}{\sin ABA'}$$

$$\frac{H}{\sin\left(\alpha + \frac{\theta}{2} - m\theta\right)} = \frac{D}{\sin[90° - (\beta + m\theta)]}$$

or

$$H = D\,\frac{\sin\left(\alpha + \frac{\theta}{2} - m\theta\right)}{\cos(\beta + m\theta)} \tag{18}$$

But from Eq. (17)

$$\beta + m\theta = \frac{\alpha + \beta}{2} + \frac{\theta}{2}$$

and

$$\alpha + \frac{\theta}{2} - m\theta = \frac{\alpha + \beta}{2}$$

$$\therefore \quad H = D\,\frac{\left(\sin\frac{\alpha + \beta}{2}\right)}{\cos\left(\frac{\alpha + \beta}{2} + \frac{\theta}{2}\right)} \tag{19}$$

For small distance, $\theta/2$ being small may be neglected and hence,

$$H = D\,\frac{\sin\left(\frac{\alpha + \beta}{2}\right)}{\cos\left(\frac{\alpha + \beta}{2}\right)} = D \tan\left(\frac{\alpha + \beta}{2}\right) \tag{20}$$

Equation (20) can be modified by replacing α and β by α_1 and β_1, the angles corrected for axis signal.

1.6 DETERMINATION OF COEFFICIENT OF REFRACTION

The coefficient of refraction m is a measure of the curvature of the line of sight and is the ratio of the radius of curvature of the earth R, to the radius of curvature of the line of sight R_s.

$$m = \frac{R}{R_s}$$

To determine the value of m, several reciprocal observations are made and a mean of all the values of coefficient of refraction is obtained.
In Fig. 1.6

$$C = \frac{\theta}{2} \tag{21}$$

and
$$\gamma = m\theta \tag{22}$$

Also,
$$\alpha + (C - \gamma) = \beta - (C - \gamma)$$

or
$$2\gamma = \alpha - \beta + 2C$$

or
$$\gamma = \frac{1}{2}(\alpha - \beta + 2C)$$

or
$$\gamma = \frac{1}{2}\left(\alpha + \beta + 2\frac{\theta}{2}\right)$$

or
$$\gamma = \frac{1}{2}(\theta + \alpha - \beta) \tag{23}$$

From Eqs. (22) and (23)

$$m\theta = \frac{1}{2}(\theta + \alpha - \beta)$$

or
$$m = \frac{\theta + \alpha - \beta}{2\theta} \tag{24}$$

☑**Note** The usual value of m obtained by this method is 0.07. It should also be noted that while defining the refraction correction linearly, it is assumed to be 1/7th of the curvature correction $\left(\frac{1}{7}\frac{D^2}{2R} = 0.14\frac{D^2}{2R} = 2m\frac{D^2}{2R}\right)$. Therefore, in such a case $2m = 0.14$, i.e. m has a value of 0.07.

Example 1.4 Two stations A and B are 1700 m apart. The observations recorded were as follows.

	Station A	*Station B*	
Height of instrument	1.39 m	1.46 m	
Height of signal	2.2 m	2.00 m	$R \sin 1'' = 30.88$ m
Vertical angle	+ 1°08′05″	− 1°06′10″	

Calculate the difference of level between A and B, the coefficient of refraction and the refraction correction.

Solution

$$\text{Vertical angle from A to B, } \alpha = 1°08'05''$$

$$\text{Axis signal correction to } \alpha = \frac{2 - 1.39}{1700} \times 206\,265$$

$$= 74.012'' \; (-)$$

$$\therefore \text{ Vertical angle corrected for axis signal, } \alpha_1 = 1°08'05'' - 74.012''$$

$$= 1°06'50.98''$$

$$\text{Vertical angle from B to A, } \beta = -\,1°06'10''$$

$$\text{Axis signal correction to } \beta = \frac{2.2 - 1.46}{1700} \times 206\,265$$

$$= 89.78''$$

$$\text{Vertical angle corrected for axis signal, } \beta_1 = 1°06'10'' + 89.78''$$

$$= 1°07'39.78''$$

$$\text{Central angle, } \theta = \frac{D}{R \sin 1''} = \frac{1700}{30.88} = 55.05''$$

$$H = \frac{D \sin\left(\dfrac{\alpha_1 + \beta_1}{2}\right)}{\cos\left[\left(\dfrac{\alpha_1 + \beta_1}{2}\right) + \dfrac{\theta}{2}\right]}$$

$$= \frac{1700 \sin\left(\dfrac{(1°06'50.98'' + 1°07'39.78'')}{2}\right)}{\cos\left[\left(\dfrac{1°06'50.98'' + 1°07'39.78''}{2}\right) + \dfrac{55.05''}{2}\right]}$$

$$= 33.263 \text{ m}$$

$$\text{Refraction correction, } \gamma = \frac{1}{2}(\theta + \alpha_1 - \beta_1)$$

$$= \frac{1}{2}(55.05 + 1°06'50.98'' - 107'39.78'')$$

$$= 3.125''$$

$$\text{Coefficient of refraction } m = \frac{\gamma}{\theta/2} = \frac{3.125}{55.05/2} = 0.113$$

Example 1.5 Two points A and B are 9 km apart with respective reduced levels as 104.5 m and 290.5 m. The signal height at A is 1.50 m and that at B is 3.10 m. The instrument height at both the stations was 1.50 m. If 30.88 m on the earth's surface subtends 1″ of arc at the earth's centre and the effect of refraction is 1/7th of that of the curvature, determine the observed angles from A to B and B to A.

Solution

$$H = 290.5 - 104.5 = 186 \text{ m}$$

We know that, $H = D \tan \phi$

or $\tan \phi = \phi = \frac{H}{D} \times 206\,265$

where $\phi = \alpha + (C - \gamma) = \beta - (C - \gamma)$

Now, $\phi = \frac{186}{9000} \times 206\,265$

$= 1°11'2.81''$

Central angle, $\theta = \frac{9000}{30.88} = 291.45''$

Curvature correction, $C = \frac{\theta}{2} = \frac{291.45}{2} = 145.725''$

Refraction correction, $\gamma = \frac{1}{7} C = \frac{145.725}{7} = 20.817''$

$\alpha = \phi - (C - \gamma) = 1°11'2.81'' - (145.725'' - 20.817'')$

$= 1°9'0.72''$

and $\beta = \phi + (C - \gamma) = 1°11'2.81'' + (145.725'' - 20.817'')$

$= 1°13'4.9''$

Since the instrument height at A is 1.5 m and signal height at B is 3.1 m

Axis signal correction is $= \frac{3.1 - 1.5}{9000} \times 206\,265 = 36.67''$

To obtain the true angle, the correction is subtracted from the observed angle of elevation. But in this example, the angle (α) computed is the true reciprocal angle. Thus, the correction has to be added to obtain the observed angle.

∴ Observed angle of elevation $= 1°9'0.72'' + 36.67''$

$= 1°9'37.39''$

Since, the instrument height at B and the signal height at A both are equal (1.5 m), no axis signal correction will be required. Therefore, the observed angle from B to A is $1°13'4.9''$.

Example 1.6 It is required to determine the elevation of a station O. Observations were made to three stations A, B and C already fixed and of known elevations. The following data was recorded.

Instrument station	*Station observed*	*Height of instrument (m)*	*Distance (m)*	*Height of signal (m)*	*Vertical angle*
	A		3600	5.6	$1°1'20''$
O	B	1.50	4700	4.1	$-53'00''$
	C		5000	4.9	$-34'10''$

The reduced level of A, B and C were 294, 159.5 and 181 m, respectively. Take m = 0.07 and $R \sin 1'' = 30.88$ m.

Solution *Axis signal correction*

Distance OA

$$\delta_1 = \frac{5.6 - 1.50}{3600} \times 206\ 265 = 234.91'' (-)$$

Distance OB

$$\delta_2 = \frac{4.1 - 1.50}{4700} \times 206\ 265 = 114.10'' (+)$$

Distance OC

$$\delta_3 = \frac{4.9 - 1.50}{5000} \times 206\ 265 = 140.26'' (+)$$

Combined correction for curvature and refraction

$$C_r = \frac{(1 - 2m)D}{2R \sin 1''}$$

Distance OA

$$C_{r_1} = \frac{(1 - 2 \times 0.07)}{2 \times 30.88} \times 3600 = 50.13''$$

Distance OB

$$C_{r_2} = \frac{(1 - 2 \times 0.07)}{2 \times 30.88} \times 4700 = 65.446''$$

Distance OD

$$C_{r_3} = \frac{(1 - 2 \times 0.07)}{2 \times 30.88} \times 5000 = 69.624''$$

Corrected vertical angles

From O to A

$$\alpha_1 = 1°1'20'' - 234.91'' + 50.13'' = 58'15.12''$$

From O to B

$$\alpha_2 = 53'.00'' + 114.10'' - 65.446'' = 53'48.654'' (-)$$

From O to C

$$\alpha_3 = 34'10'' + 140.26'' - 69.624'' = 35'20.936'' (-)$$

Difference of levels of stations

$$H = D \tan \alpha$$

$$H_1 = 3600 \tan 58'15.12'' = + 61.007 \text{ m}$$

$$H_2 = 4700 \tan 53'48.654'' = - 73.575 \text{ m}$$

$$H_3 = 5000 \tan 35'20.936'' = - 51.414 \text{ m}$$

R.L. of O

In first case = 294 – 61.007 = 232.993 m

In second case = 159.5 + 73.575 = 233.075 m

In third case = 181.0 + 51.414 = 232.414 m

∴ Mean elevation of O = 232.827 m

Exercises

1.1 Define the coefficient of refraction. Explain how its value can be obtained by simultaneous reciprocal observations.

1.2 Describe the difference between the techniques of reciprocal levelling and reciprocal trigonometrical levelling, and discuss the conditions in which each is most effectively used.

1.3 Derive an expression for the difference of level between two points A and B a distance D apart, with the vertical angle as the angle of elevation from A to B. The height of the, instrument at A and that of the signal at B are equal.

1.4 It is required to find the difference of elevation between two stations A and B 9290 m apart. The angle of elevation from A to B was observed with a theodolite as 2°06′18″ and the height of the instrument was 1.25 m. The height of signal at station B was 3.96 m. Take the value of R sin 1″ as 30.88 and coefficient of refraction as 0.07. Also, find the R.L. of station B if the R.L. of station A is 300 m.

[*Ans*. 344.59 m, 644.59 m]

1.5 The distance between two stations A and B is 6370 m. The station B was 200 m above the station A. Calculate the angles observed from A and B. Assume that the instrument and signal heights to be equal and the effect of refraction as 1/7th of that of the curvature. The radius of earth is 6370 km.

[*Ans*. 1°46′27.73″, 1°49′24.53″]

1.6 Two stations A and B are 16.44 km apart. The following data were recorded: Instrument at A, angle of depression to B = 3′42″

Instrument at B, angle of depression to A = 2′04″

Height of instrument at A	= 1.42 m
Height of instrument at B	= 1.42 m
Height of signal at A	= 5.53 m
Height of signal at B	= 5.53 m
R sin 1″	= 30.88 m

Find the difference in level between A and B and the coefficient of refraction at the time of observation.

[*Ans*. 3.91 m, 0.0784]

1.7 The distance between two stations A and B was 3489.96 m. Find out the reduced level of the station B, if the R.L. of A was 950.75 m. The following observations were recorded:

	Station A	*Station B*
Height of instrument	1.433 m	1.463 m
Height of signal	4.572 m	3.962 m
Vertical angle	+ 1°52′4″ (to B)	− 1°48′20″ (to A)

Take the value of R sin 1″ as 30.88.

[*Ans*. 1062.932 m]

1.8 The following data refer to the elevations of the ground stations of a triangle ABC in a trigonometrical survey. Find the closing error and the R.L.s of B and C if the R.L. of A is 1600 m.

(i) Vertical angle from A to B = + 1°20′20″
Vertical angle from B to A = – 1°12′24″
Weight 2
Height of instrument at A = 1.50 m
Height of signal at A = 4.92 m
Height of instrument at B = 1.47 m
Height of signal at B = 4.44 m
Distance AB = 4777.8 m

(ii) Vertical angle from B to C = – 49′24″
Vertical angle from C to B = + 55′12″
Weight 3
Height of instrument at B = 1.44 m
Height of signal at B = 4.71 m
Height of instrument at C = 1.41 m
Height of signal at C = 5.52 m
Distance BC = 4068.2 m

(iii) Vertical angle from C to A = – 47′12″
Weight 1
Height of instrument at A = 1.44 m
Height of signal at C = 3.96 m
Distance AC = 3187.5 m

[*Ans.* 2410 m, 1706.803, 1644.79 m]

1.9 Two stations A and B were a distance 1800.50 m apart. Reciprocal observations were made to determine the difference of level between them. The following data were recorded:

Height of instrument at A = 1.463 m
Height of instrument at B = 1.457 m
Height of signal at A = 1.647 m
Height of signal at B = 1.762 m
Vertical angle from A to B = + 1°42′2″
Vertical angle from B to A = – 1°41′46″

Calculate also the height of B above A, using the vertical angle from A only. Assume the coefficient of refraction as 0.07. Prove the formula used.

[*Ans.* 53.32 m, 53.356 m]

Objective-type Questions

1.1 The process of determining the elevations of stations from vertical angles and geodetic lengths at mean sea level is known as
(a) levelling (b) trigonometric levelling
(c) triangulation (d) hypsometry

1.2 Trigonometric levelling by reciprocal observations
(a) eliminates error due to uncertain refraction.
(b) is essentially done in the early morning.
(c) proves to be more accurate than spirit levelling.
(d) all are correct

1.3 In trigonometric levelling the correction due to refraction and curvature are determined
(a) in linear measure
(b) in angular measure
(c) either (a) or (b)
(d) graphically

1.4 Which one is a correction to be applied in trigonometric levelling
(a) correction for dip
(b) correction for semi-diameter of sun
(c) axis signal correction
(d) parallax correction

1.5 In trigonometric levelling the correction is subtracted to the observed vertical angle if it is for
(a) axis signal
(b) combined refraction and curvature
(c) both (a) and (b)
(d) none

Answers to Objective-type Questions

1.1 (b) 1.2 (a) 1.3 (b) 1.4 (c) 1.5 (b)